MIDTERM 2 STUDY GUIDE

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<u>Know how to:</u>

- Given the graph of a function, sketch the graph of its derivative (2.8.5)
- Given 3 graphs, determine which one is the graph of f, f', f'' (2.8.41)
- Find the derivative of a function using the definition of the derivative (2.8.21, 2.8.28)
- **Know how to find the derivative of a function**
  (1) Using the power rule ($(x^n)' = n(x^{n-1})$), valid for **all** nonzero numbers n, even $\frac{1}{2}$, or $\sqrt{2}$) (3.1.6, 3.1.8)
  (2) Using the sum rule ($(f+g)' = f'+g'$) and the constant multiple rule ($(cf)' = c(f')$) (3.1.23)
  (3) Using $(e^x)' = e^x$, as well as $(a^x)' = ln(a) \cdot a^x$ and $(ln(x))' = \frac{1}{x}$ (3.1.17, 3.1.32, 3.6.3, 3.6.16, 3.6.18)
  (4) Using the product rule ($(fg)' = (f')g + f(g')$) and the quotient rule ($(\frac{f}{g})' = \frac{(f')g - f(g')}{g^2}$) (3.2.15, 3.2.18, 3.2.24, 3.2.26)
  (5) Using derivatives of trigonometric functions ($(cos)' = -sin$, $(sin)' = cos$, $(tan)' = sec^2$) (3.3.10, 3.3.12, 3.3.24)
  (6) **Using the chain rule** ($(f \circ g)'(x) = g'(x) \cdot f'(g(x))$) (3.4.5, 3.4.13, 3.4.29, 3.4.42, 3.4.46, 3.4.50, 3.4.71)
  (7) Using implicit differentiation (3.5.11, 3.5.18, 3.5.27, 3.5.36, **3.5.54**)
  (8) Using logarithmic differentiation (3.6.30, 3.6.41, 3.6.42, 3.6.50)
  Note: Be sure to know how to combine those methods, and **THINK** about your problem before you tackle it!
- Find the equation of the tangent line to a graph at a point (3.1.35, 3.2.32, 3.3.24, 3.4.54, 3.5.28, 3.6.33)
- Find the equation of the normal line to a graph at a point (3.1.35)
- Find numbers where a tangent line to a graph is horizontal, or parallel to a given line (3.1.51, 3.1.55)
- Find n^{th} derivatives of functions (3.1.62)
- Solve word problems using derivatives (3.3.5, 3.3.37, 3.4.82, 3.7.10, 3.7.18), basically, derivatives represent rates of change
- Solve problems using $\lim_{x \to 0} \frac{sin(x)}{x} = 1$ (3.3.39, 3.3.46, **3.3.51**)
- Solve the differential equation $y' = ky$ (i.e. $y = Ce^{kx}$), and use that formula in real-life situations (3.8.3, 3.8.5)
- Using $y' = ky$ and other information, find, for example C, or k, or $y(something)$ or the half-life of an element (3.8.9, 3.8.10)
- Solve problems using Newton's law of cooling (3.8.13, 3.8.15)
- **Solve related rates problems** (3.9.13, 3.9.17, 3.9.18, 3.9.24, 3.9.36, 3.9.44)

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- Find the linear approximation of a function f at a given point a (3.10.1, 3.10.3)
- Use a linear approximation to estimate a given number, e.g. $\sqrt{98}$ (3.10.25, 3.10.27, 3.10.38)
- Find the differential dy of a function, and evaluate dy for given values of x and dx (3.10.15)
- Solve word problems using differentials (3.10.36)
- Given a graph, state the absolute/local maximum/minimum values of a function (4.1.5)
- Sketch the graph of a function with given properties having to do with local/absolute max/min (4.1.7, 4.1.10, 4.1.11, 4.1.14)
- Find the critical numbers of a given function (4.1.33, 4.1.35, 4.1.42, 4.1.44)
- Find the absolute max/min of a given function on a given closed interval (4.1.47, 4.1.51, 4.1.55, 4.1.61, 4.1.62, 4.1.63)
- **Use the Mean Value Theorem to:**
  - Show that an equation has at most one/two roots (4.2.19, 4.2.20, 4.2.22)
  - Show that an equation has **exactly** one root, using in addition the Intermediate Value Theorem (4.2.17, 4.2.18)
  - Estimate the value of a function (look at 4.2.23, 4.2.24, 4.2.25, 4.2.26)
  - Solve other problems using the mean value theorem (4.2.28, 4.2.29, 4.2.35, 4.2.36)
- Show that two functions f and g are equal by differentiating them and plugging in one value for x (4.2.32, 4.2.33)
- Show that $f < g$ by considering $h = f - g$, and differentiating h (4.2.27, 4.3.74)
- Find intervals on which f is increasing/decreasing, finding the local max/min of f, as well as intervals of concavity and inflection points, given a graph or given a formula (4.3.8, 4.3.11, 4.3.13)
- Find a local max/min using the First and/or Second derivative tests (4.3.19)
- Sketch the graph of a function with given properties having to do with first/second derivatives (4.3.24, 4.3.26)
- Calculate limits using l'Hopital's rule **or another method!!!** (4.4.9, 4.4.14, 4.4.15, 4.4.29, 4.4.49, 4.4.59, 4.4.64, 4.4.72, as well as all the problems between 5 and 64 in section 4.4 if you need more practice!)
- **Sketch the graph of a given function** f, using the DISAIC-method, making sure to label your graph (4.5.5, 4.5.15, 4.5.21, 4.5.38, 4.5.52, 4.5.56):
  - **D**omain
  - **I**ntercepts (x- and y- intercepts)
  - **S**ymmetry (even, odd, periodic)
  - **A**symptotes (horizontal, vertical, slant)
  - **I**ncreasing/Decreasing/Local Max/Min (Calculate f')
  - **C**oncave Up/Down/Inflection Points (calculate f'')
- Show that a given line is the slant asymptote to f at ∞ or $-\infty$ (4.5.67, 4.5.68)
- Find the slant asymptotes to f at ∞ or $-\infty$ (4.5.64, 4.5.65)
  Note: If you have trouble with this, check out the 'Slant Asymptotes'-handout sent out on Thursday night
- **Solve optimization problems** (4.7.3, 4.7.16, 4.7.17, 4.7.19, 4.7.22, 4.7.25, 4.7.37, 4.7.46, 4.7.53, 4.7.67)
- Apply Newton's method once or twice on a given function f with a given initial value of x_0 (4.8.5)

Also, know how to **derive** the following

- The derivative of $\csc(x)$, $\sec(x)$, $\cot(x)$ (this is just the quotient rule)
- The derivative of $f^{-1}(x)$ in terms of $f'(x)$ (3.6.67)
- The derivative of $\ln(x)$ **and** the derivative of $\ln(|x|)$
- The derivative of $\cos^{-1}(x)$, $\sin^{-1}(x)$, $\tan^{-1}(x)$
- $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$
- $\lim_{x\to 0} \frac{1-\cos(x)}{x} = 0$
- $e = \lim_{x\to 0}(1+x)^{\frac{1}{x}}$
- Second derivatives using the chain rule (3.4.95)
- The solution of $T' = k(T - T_s)$ (i.e. Newton's law of cooling)
- If $f'(x) = 0$ for all x in (a, b), then f is constant on (a, b)
- If $f'(x) > 0$ for all x in (a, b), then f is increasing on (a, b)
- If $f'(x) > 0$ for all x in (a, b) **except for a point** c, and f is differentiable on (a, b) , then f is increasing on (a, b) (4.3.71)
- If $f'(x) = g'(x)$ for all x in an interval (a, b), then $f(x) = g(x) + c$, where c is a constant

Finally, know how to define/state the following (you **don't** need to know how to prove these)

- The derivative of a function f at a
- f is differentiable at a, or on I (I is an interval)
- The sum, product, quotient, chain rules (**with all the assumptions**)
- e (the **new** definition: e is the number such that $\lim_{h\to 0} \frac{e^h-1}{h} = 1$)
- The linear approximation of f at a
- The differential dy (i'ts just $dy = f'(x)dx$)
- f has an absolute maximum/minimum at c
- f has a local maximum/minimum at c
- **The extreme value theorem**
- **Fermat's theorem**
- A critical number c of f
- **Rolle's theorem**
- **The mean value theorem**
- Increasing/Decreasing test
- **The first derivative test**
- Concave up/Concave down (**WARNING:** The definition used in the book is wrong! Use Prof. Christ's definition!!!)
- Inflection point
- **The second derivative test**
- L'Hopital's rule
- Slant asymptote at ∞ or $-\infty$
- **Newton's method** (just say $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$)